Lebesque's Dominated Convergence Theorem (Theorem 2.4.2)

Suppose the function $h(x,y)$ is continuous at y_0 for each x, and there exists a function $g(x)$ satisfying:

i) $\left|h(x,y)\right| \le g(x)$ for all x and y

ii) $\int_{-\infty}^{\infty} g(x)dx < \infty$

Then $\lim_{y\to y_0} \int_{-\infty}^{\infty} h(x,y)dx = \int_{-\infty}^{\infty} \lim_{y\to y_0} h(x,y)\,dx$

Uniform convergence and integration.

Assume $\{f_n\} \to f$ uniformly on [a,b] and that the Riemann integral of f_n exist for n=1, 2, … . Then the Riemann integral of f exists and

$\int_a^x f_n(t)\,dt \to \int_a^x f(t)dt$ uniformly on [a,b].

Uniform convergence and derivation

Assume for $\{f_n\}$ that $\{f_n^{'}\}$ is continuous in [a,b]. Assume there exists at $x_0 \in [a,b]$ such that $\{f_n(x_o)\}$ converges and that $\{f_n^{'}\}$ converges uniformly in [a,b]. Then $\{f_n\}$ converges uniformly towards a function f and in [a,b] and

$$\frac{d}{dx}\left(\lim_{n\to\infty} f_n(x)\right) = f^{'}(x) = \lim_{n\to\infty} f_n^{'}(x)$$

Theorem 2.4.8

Suppose that the series $\sum_{x=0}^{\infty} h(x,\theta)$ converges for all $\theta \in (a,b)$ and

i) $\frac{d}{d\theta} h(x,\theta)$ is continuous in θ for each x

ii) $\sum_{x=0}^{\infty} \frac{d}{d\theta} h(x,\theta)$ converges uniformly on every closed bounded interval subinterval of (a,b)

Then $\frac{d}{d\theta} \sum_{x=0}^{\infty} h(x,\theta) = \sum_{x=0}^{\infty} \frac{d}{d\theta} h(x,\theta)$

Overview of some natural occurring distributions

Independent trials Register: $\mathrm{A}/\mathrm{A}^{\mathrm{c}}$ $P(A)=p$	Events in disjoint timeintervals are independent $P(\text{One event in } \Delta\text{t})=\lambda\Delta\text{t}+o(\Delta\text{t})$ $P(\text{More than one event in } \Delta\text{t})=o(\Delta\text{t})$
X=number of times A occurs in n trials $P(X=x)=\binom{n}{x}p^{x}(1-p)^{n-x}, x=0,1,\ldots,n$	X=number of times A occur in [0,t] $P(X=x)=\dfrac{(\lambda t)^{x}e^{-\lambda t}}{x!},\ x=0,1,2,\ldots$
X=number of trials until A occurs for the first time $P(X=x)=(1-p)^{x-1}p,\ \mathrm{x}=1,2,\ldots$	X= time until A occurs for the first time $f_X(x)=\begin{cases}\lambda e^{-\lambda x}, & \mathrm{x}>0\\ 0, & \text{otherwise}\end{cases}$
X=number of trials until A occurs for the r-th time $P(X=x)=\binom{x-1}{r-1}p^{r}(1-p)^{x-r},\ \mathrm{x}=r,r+1,\ldots$	X=time until A occurs the r-th time $f_X(x)=\begin{cases}\dfrac{\lambda^{r}}{\Gamma(r)}x^{r-1}e^{-\lambda x}, & \mathrm{x}>0\\ 0, & \text{otherwise}\end{cases}$